

8. E. I. Antonov, V. E. Il'in, E. A. Kolenko, et al., Devices for Cooling Radiation Detectors [in Russian], Mashinostroenie, Leningrad (1969).
9. E. A. Kolenko, Thermoelectric Cooling Devices [in Russian], Nauka, Moscow (1967).
10. G. I. Voronin (ed.), Low Temperature Thermal Tubes [in Russian], Mashinostroenie (1976).
11. D. Lucas (ed.), Heat Exchange and Thermal Regimes of Space Vehicles [Russian translation], Mir, Moscow (1974).
12. G. N. Dul'nev, B. V. Pol'shchikov, and A. Yu. Potyailo, "Design of an algorithm for heirarchical modelling of heat exchange processes in complex radio equipment," Radiotekhnika, 34, No. 11, 49-54 (1979).
13. O. Aga, G. N. Dul'nev, and B. V. Pol'shchikov, "Thermal modeling of electrotechnical devices," Inzh.-Fiz. Zh., 40, No. 6, 1062-1069 (1981).
14. G. N. Dul'nev, A. Yu. Potyailo, and E. D. Ushakovskaya, "A numerical-analytical method for calculation of temperature fields in complex objects," in: Heat-Mass Exchange IV [in Russian], Vol. 9, Minsk (1980), pp. 56-61.
15. G. N. Dul'nev and N. N. Tarnovskii, Thermal Regimes of Electronic Apparatus [in Russian], Énergiya, Leningrad (1971).
16. G. N. Dul'nev, "The local effect principle in thermal conductivity theory," Tr. LITMO-L, No. 37, 17-24 (1959).
17. B. V. Pol'shchikov and E. D. Ushakovskaya, "Methods for replacing one-dimensional regions with distributed heat sources," Izv. Vyssh. Uchebn. Zaved., Priborostr., 23, No. 11, 92-96 (1981).
18. L. A. Savintseva, E. D. Strelova, and N. N. Tarnovskii, "Use of four-port theory for analysis of temperature fields in complex objects," Deposited VINITI, No. 4191-72, Inzh.-Fiz. Zh., 23, No. 1, 163-164 (1972).
19. A. V. Lykov and B. M. Berkovskii, Convection and Thermal Waves [in Russian], Énergiya, Moscow (1974).
20. A. V. Lykov, "Conjugate convective heat transfer problems," Int. J. Heat Mass Transfer, 17, No. 2, 257-265 (1974).
21. M. A. Mikheev and I. M. Mikheeva, Fundamentals of Heat Transfer [in Russian], Énergiya, Moscow (1977).
22. G. N. Rokhlin, Gas Discharge Light Sources [in Russian], Énergiya, Moscow-Leningrad (1966).
23. A. V. Lykov, Theory of Thermal Conductivity [in Russian], Vysshaya Shkola, Moscow (1967).

METHOD OF CALCULATING TEMPERATURE PROFILES

IN TWO-PHASE ANNULAR FLOWS

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On the basis of the energy equation, a method of calculating temperature profiles in a liquid film and in the surrounding gas flow is given.

Experimental measurement of local temperatures in the cross section of a thin liquid film is technically very difficult, and does not provide the required accuracy. In this connection, analytical methods of investigation are expedient.

Temperature profiles in two-phase films are determined by the physical properties of the components, the conditions at the inlet, and at the phase boundaries, and also the hydrodynamics of the flow, the parameters of which are unknown [1]. Thus, the problem reduces to solving the energy equation [2]

$$\omega_l \frac{\partial T_l}{\partial x} = - (a_l + a_{lr}) \frac{\partial^2 T_l}{\partial y^2} + \left(\frac{a_l + a_{lr}}{R-y} - \frac{\partial a_{lr}}{\partial y} \right) \frac{\partial T_l}{\partial y} \quad (1)$$

with the initial conditions

$$T_1 = T_{10}(y), \quad T_2 = T_{20}(y) \quad \text{for } x = 0 \quad (2)$$

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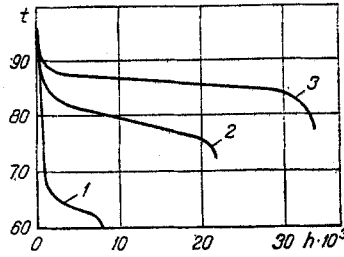


Fig. 1. Temperature distribution $t(^{\circ}\text{C})$ in the cross section of a water film at different Reynolds numbers: 1) $\text{Re}_1 = 3560$; $\text{Re}_2 = 33600$; $t_c = 86^{\circ}\text{C}$; 2) 9100, 35340, 92; 3) 15040, 34860, 98.

and boundary conditions

$$T_1 = T_c(x) \text{ for } y = 0, T_2 = T_R(x) \text{ for } y = R, \quad (3)$$

$$T_1 = T_{1\delta}(x), T_2 = T_{2\delta}(x) \text{ for } y = \delta. \quad (4)$$

In Eq. (1), the turbulent thermal diffusivities of the components $a_{\tau T}$ are known functions of the coordinates [3].

After introducing the notation $b = 0.25u\tau\text{Re}\tau$, $k = m/(1 - \epsilon)$, $f = (h - \epsilon)/h$, $q = \epsilon/h$, $n = bh$, $m = a_{\tau} + a_{\tau T}$ and the substitution

$$Z_1 = T_1 - fT_c - qT_{1\delta} \quad (5)$$

the equation for the liquid component reduces to the form

$$\frac{\partial Z_1}{\partial \eta} + mn \frac{\partial^2 Z_1}{\partial \epsilon^2} - n \left(k - \frac{\partial a_{1\tau}}{\partial \epsilon} \right) \frac{\partial Z_1}{\partial \epsilon} = b(T_{1\delta} - T_c) \left(k - \frac{\partial a_{1\tau}}{\partial \epsilon} \right) - f \frac{\partial T_c}{\partial \eta} - q \frac{\partial T_{1\delta}}{\partial \eta} \quad (6)$$

with the homogeneous boundary and initial conditions

$$Z_1 = Z_{10} = T_{10} - fT_{c0} - qT_{1\delta 0} \text{ when } \eta = 0. \quad (7)$$

The solution of the homogeneous Eq. (6) is found by the Fourier method, according to which $Z_{11} = M(\eta)N(\epsilon)$. The function $N(\epsilon)$ is found by the collocation method from the expression

$$N = \sum_{i=1}^n c_i \epsilon^i (h - \epsilon). \quad (8)$$

If the coefficients c_i with $n = 3$ are determined at fixed collocation points ϵ_i (for example, $\epsilon_i = 0.25h, 0.5h, 0.75h$), and the following notation is introduced

$$\varphi_{1i} = 2(h - 2\epsilon_i) P_i, \varphi_{2i} = [2(h - 3\epsilon_i) - \epsilon_i(2h - 3\epsilon_i)] P_i,$$

$$\varphi_{3i} = \epsilon_i [6(h - 2\epsilon_i) - \epsilon_i(3h - 4\epsilon_i)] P_i,$$

$$P_i = (1 - \epsilon_i)^{-1} - m^{-1} \left(\frac{\partial a_{1\tau}}{\partial \epsilon} \right)_{\epsilon=\epsilon_i}, \psi_{ni} = \epsilon_i^n (h - \epsilon_i) (mn)^{-1} \quad (i, n=1, 2, 3),$$

$$c_1 = \frac{(\varphi_{32} - \psi_{32}\lambda_{1j}^2)(\varphi_{11} + \psi_{11}\lambda_{1j}^2) - (\varphi_{12} + \psi_{12}\lambda_{1j}^2)(\varphi_{31} - \psi_{31}\lambda_{1j}^2)}{(\varphi_{22} - \psi_{22}\lambda_{1j}^2)(\varphi_{11} + \psi_{11}\lambda_{1j}^2) - (\varphi_{12} + \psi_{12}\lambda_{1j}^2)(\varphi_{21} - \psi_{21}\lambda_{1j}^2)},$$

it is simple to obtain the result

$$Z_{11} = \sum_{j=1}^4 c(\lambda_{1j}^2) \epsilon f h \left[\frac{(\varphi_{21} - \psi_{21}\lambda_{1j}^2) c_1 + \varphi_{31} + \psi_{31}\lambda_{1j}^2}{\varphi_{11} + \psi_{11}\lambda_{1j}^2} + \epsilon(c_1 + \epsilon) \right] \exp(-\lambda_{1j}^2 \eta) = \sum_{j=1}^4 c(\lambda_{1j}^2) Z_{11}^*. \quad (9)$$

The eigenvalues λ_{1j}^2 in Eq. (9) are positive roots ($j = 4$) of the equation

$$(\varphi_{13} + \psi_{13}\lambda_1^2)(\varphi_{21} - \psi_{21}\lambda_1^2) [(\varphi_{32} - \psi_{32}\lambda_1^2)(\varphi_{11} + \psi_{11}\lambda_1^2) - (\varphi_{12} + \psi_{12}\lambda_1^2)(\varphi_{31} - \psi_{31}\lambda_1^2)] - (\varphi_{13} + \psi_{13}\lambda_1^2)(\varphi_{31} - \psi_{31}\lambda_1^2) \times$$

$$\begin{aligned} & \times [(\varphi_{22} - \psi_{22}\lambda_1^2)(\varphi_{11} + \psi_{11}\lambda_1^2) - (\varphi_{12} + \psi_{12}\lambda_1^2)(\varphi_{21} - \psi_{21}\lambda_1^2)] - \\ & - (\varphi_{33} - \psi_{33}\lambda_1^2)(\varphi_{11} + \psi_{11}\lambda_1^2) [(\varphi_{22} - \psi_{22}\lambda_1^2)(\varphi_{11} + \psi_{11}\lambda_1^2) - \\ & - (\varphi_{12} + \psi_{12}\lambda_1^2)(\varphi_{21} - \psi_{21}\lambda_1^2)] = 0. \end{aligned} \quad (10)$$

The coefficients $c(\lambda_{1j}^2)$ are determined from a system of equations formed from Eq. (9) and the initial conditions in Eq. (7) with $\eta = 0$.

$$Z_{10k} = \sum_{j=1}^4 c(\lambda_{1j}^2) a_{kj}, \quad (11)$$

where Z_{10k} , a_{kj} are, respectively, the functions Z_{1k} and Z_{11}^* written for the fixed points ε_k ($k = 1-4$) when $\eta = 0$. The system of equations obtained here leads to the result

$$c(\lambda_{1j}^2) = D_{1j} D_1^{-1}. \quad (12)$$

In this expression D_1 is the determinant of the system in Eq. (11), while D_{1j} is the determinant formed from D_1 by replacing its j -th column by a column composed of Z_{10k} .

To find the particular solution of the inhomogeneous Eq. (6) satisfying the zero initial and boundary conditions, this solution and the function on the right-hand side of Eq. (6) are written in the form of a series expansion in terms of the eigenfunctions. As a result, taking account of Eq. (5), the following expression may be obtained for the temperature profile in the film

$$T_1 = \frac{h-\varepsilon}{h} T_c + \frac{\varepsilon}{h} T_{10} + \sum_{j=1}^4 \varepsilon(h-\varepsilon) \left[\varepsilon(c_1 + \varepsilon) + \frac{(\varphi_{21} - \psi_{21}\lambda_{1j}^2)c_1 + \varphi_{31} - \psi_{31}\lambda_{1j}^2}{\varphi_{11} + \psi_{11}\lambda_{1j}^2} \right] \left[c(\lambda_{1j}^2) + \int_0^\eta J_j(\eta) d\eta \right] \exp(-\lambda_{1j}^2\eta), \quad (13)$$

where $J_j(\eta)$ are the coefficients of the series expansion of the right-hand side of Eq. (6) in terms of the eigenfunctions.

Solving Eq. (1) analogously for the gaseous component, with the boundary conditions in Eqs. (2)-(4), the temperature profile in the gas is obtained in the form

$$\begin{aligned} T_2 = & T_{20}(1-\varepsilon)(1-h)^{-1} + T_R(\varepsilon-h)(1-h)^{-1} + \sum_{j=1}^4 (\varepsilon-h)(1-\varepsilon) \times \\ & \times \left[\frac{(\varphi'_{21} - \psi'_{21}\lambda_{2j}^2)c'_2 + \varphi'_{31} - \psi'_{31}\lambda_{2j}^2}{\varphi'_{11} + \psi'_{11}\lambda_{2j}^2} + (1-\varepsilon)(c'_2 + 1-\varepsilon) \right] \left[c_2(\lambda_{2j}^2) + \int_0^\eta J_{2j}(\eta) d\eta \right] \exp(-\lambda_{2j}^2\eta). \end{aligned} \quad (14)$$

The values of all the coefficients in this formula are calculated by the same methods as for the corresponding quantities in the liquid component.

The unknown temperatures of the components at the interface $T_{1\delta}$ and $T_{2\delta}$ may be determined from the condition of heat-flux continuity

$$Q_{1\delta}(\eta) = Q_{2\delta}(\eta) + \alpha [T_{1\delta}(\eta) - T_{2\delta}(\eta)]. \quad (15)$$

This condition leads to a first-order linear differential equation in terms of the temperatures to be determined, with a solution of exponential type. The heat fluxes $Q_{1\delta}$ and $Q_{2\delta}$ in the liquid and the gas at the surface of the film are found using Eqs. (13) and (14). However, additional information on the heat-transfer coefficient α at the phase interface is necessary here; in the general case, α may be a function of the longitudinal coordinate. Therefore, it is expedient to write $T_{1\delta}$ in the form $T_{1\delta} = A\zeta \exp(B\zeta\eta)$, which leads finally to the expression

$$T_{1\delta} = T_{1\delta 0} \exp[(\ln T_{1\delta 0} - \ln T_{1\delta 0}) L^{-1}\eta]. \quad (16)$$

Using Eq. (16), it is simple to take the integrals appearing in Eqs. (13) and (14).

The solution of Eqs. (13) and (14), taking Eq. (16) into account, is simple to obtain on a computer. As an example, calculational temperature profiles in a film of water moving together with an air flow in a tube of diameter 30 mm corresponding to the experimental conditions of [4, 5] are shown in Fig. 1. As is evident from Fig. 1, the thickness of the wall boundary layer in the film where there is a linear temperature-distribution law, is small, and decreases with increase in the Reynolds number of the film. With increase in liquid flow rate, the turbulization of the film (wave amplitude) increases, which decreases the temperature gradient, despite a certain smoothing influence of the gas glow [5].

The results obtained at small gas velocities are in qualitative agreement with the existing data for single-phase flow [6]. In a two-phase film flow, the gas component has only slight influence on the temperature distribution in the film, but, by decreasing the thickness of the film, increases the temperature gradient within it.

NOTATION

r, x , radial and axial cylindrical coordinates; H , length of the section of tube considered; R , tube radius; $y = R - r$; δ , film thickness; T , temperature; w , velocity; α , thermal diffusivity; $Re_1 = 4\bar{w}\delta\nu_1^{-1}$; $Re_2 = 2\bar{w}_2R\nu_2^{-1}$; $\alpha^* = \alpha\nu^{-1}$; $L = HR^{-1}$; $\eta = xR^{-1}$; $h = \delta R^{-1}$; $\epsilon = yR^{-1}$; $u = \bar{w}\bar{w}^{-1}$. Indices: $l = 1$, liquid; $l = 2$, gas; c, δ, O, L, R , values at the tube wall, phase interface, tube inlet, tube outlet, and flow axis; a bar over a symbol denotes the mean value.

LITERATURE CITED

1. A. D. Berkuta, I. M. Fedotkin, M. N. Chepurnoi, and V. E. Shnaider, "Laws of two-phase annular flow in vertical tubes," Dokl. Akad. Nauk Ukr. SSR, No. 9, 855-857 (1976).
2. D. Hewitt and N. Hall-Taylor, Annular Two-Phase Flow [Russian translation], Energiya, Moscow (1974).
3. V. M. Ievlev, Turbulent Motion of High-Temperature Continuous Media [in Russian], Nauka, Moscow (1975).
4. I. M. Fedotkin, M. N. Chepurnoi, V. E. Shnaider, et al., "Hydrodynamics of descending turbulent flow of liquid and gas films," Inzh.-Fiz. Zh., 17, No. 2, 326-329 (1974).
5. I. M. Fedotkin, M. N. Chepurnoi, V. E. Shnaider, et al., "Film thickness in apparatus with descending direct flows," Pishch. Prom., No. 22, 75-78 (1976).
6. Yu. M. Tananaiko and E. G. Vorontsov, Methods of Calculating and Investigating Film Processes [in Russian], Tekhnika, Kiev (1975).

APPLICATION OF A CONSERVATIVE DIFFERENCE EQUATION

TO DETERMINE NONSTATIONARY HEAT FLUXES

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The possibility of practical utilization of a conservative difference equation of heat conduction, obtained by an integrointerpolation method, for the automatic determination of nonstationary fluxes is analyzed.

The continuous automatic determination of the heat flux is an important problem in the study of nonstationary heat transfer, particularly for the determination of the thermophysical properties (TPP) of substances.

Sufficiently complex algorithms that can be realized only by using digital computer facilities are utilized in existing high-accuracy methods of determining the heat flux density described in [1, 2]. However, utilization of analog apparatus for this purpose is more logical from the viewpoint of fast-response and the simplicity of technical realization. Such an approach to the determination of the heat flux density, based on the solution of the inverse heat conduction problem (IHCP) by using analog facilities was apparently proposed first in [3], but it is impossible to acknowledge the method mentioned as correct.

The simplest method of determining the heat flux, which permits its measurement in analog form during experiment, is based on the following interpolation of the Fourier equation [4]:

$$q(0, \tau) = \frac{\lambda f(x, \tau) \Big|_{x=L}}{L} \quad (1)$$

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